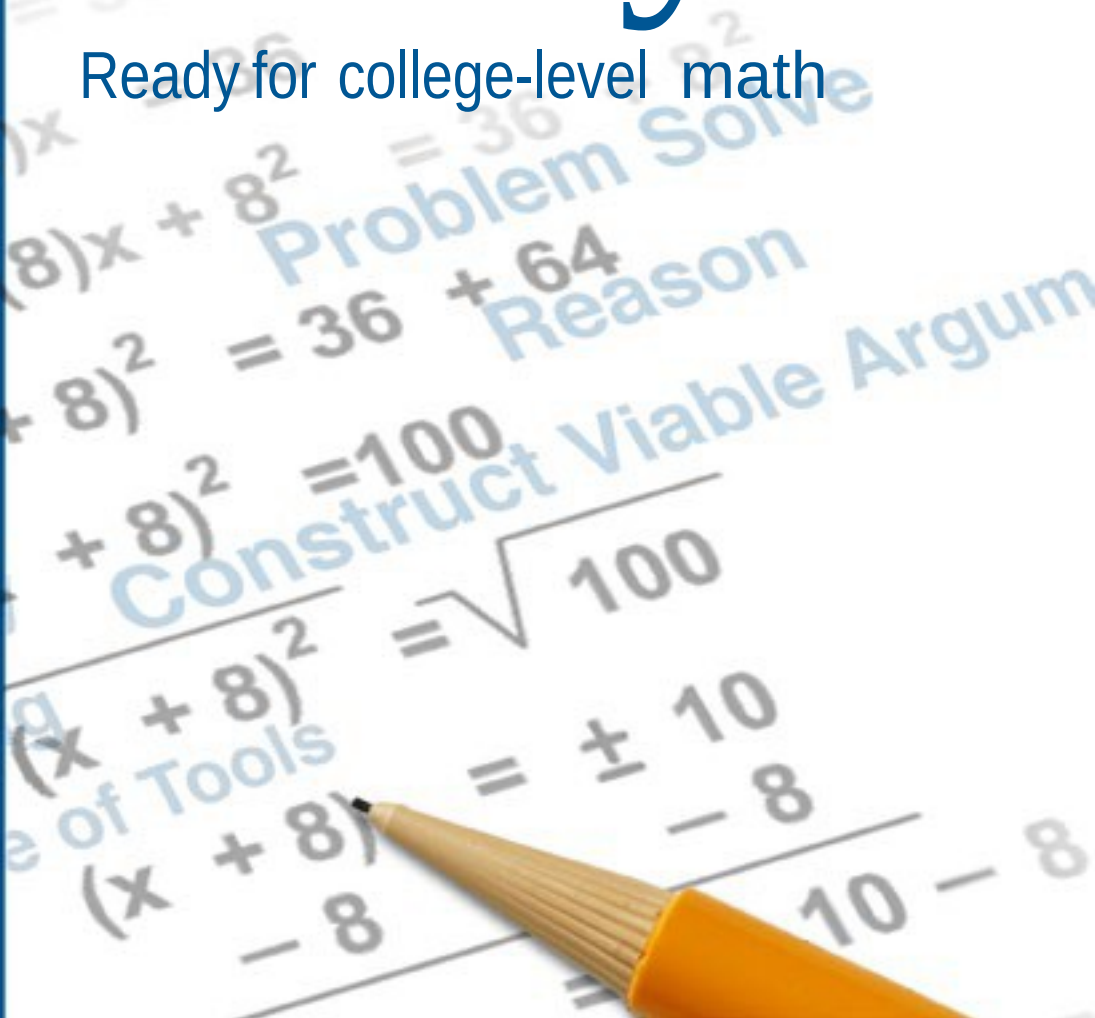
SREB

SREB Readiness Courses

Transitioning to college and careers

Math Ready

Ready for college-level math



Southern Regional Education Board

592 Tenth Street, NW
Atlanta, GA 30318
(404) 875-9211

www.sreb.org

Expressions
Overview

Purpose

This unit was designed to solidify student conception of expressions while providing the students with an opportunity to have success early in the course. The recurring theme integrated in this unit focuses on engaging students using and expanding the concepts found within purposefully chosen activities. Through guided lessons, students will manipulate, create, and analyze algebraic expressions and look at the idea of whether different sets of numbers are closed under certain operations. The writing team selected content familiar to the students in this unit to build student confidence and acclimate students to the course's intended approach to instruction.

Essential Questions:

When is estimation appropriate?

How can you extend the properties of operations on numerical expressions to algebraic expressions?

How can you apply the properties of operations to generate equivalent expressions?

How can you determine when two algebraic expressions are equivalent or not?

How can rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related?

How can you use the structure of an expression to identify ways to rewrite it?

Common Core State Standards:

Quantities

Reason quantitatively and use units to solve problems.

- N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Seeing Structure in Expressions

Interpret the structure of expressions.

- A.SSE.1: Interpret expressions that represent a quantity in terms of its context.
- A.SSE.2: Use the structure of an expression to identify ways to rewrite it.

  Write expressions in equivalent forms to solve problems.
- A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Interpreting Functions

Analyze functions using different representations.

- F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Prior Scaffolding Knowledge / Skills:

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

- *6.EE.1: Write and evaluate numerical expressions involving whole-number exponents.*
- *6.EE.2a-c: Write, read, and evaluate expressions in which letters stand for numbers.*
- *6.EE.3: Apply the properties of operations to generate equivalent expressions.*
- *6.EE.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).*

Use properties of operations to generate equivalent expressions.

- *7.EE.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.*
- *7.EE.2: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.*

The Number System

Apply and extend previous understandings of operations with fractions.

- *7.NS.1-3: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Lesson Progression Overview:

Lesson Big Idea	Lesson Details	Content Standards	Standards for Mathematical Practice
Lesson 1: Numbers and Estimation	Students will be introduced to the course using an estimation activity that will be used to develop conception of numbers and reinforce numeral operation fluency. It is also an entry activity into the course showcasing the explicit incorporation of math practices including problem solving, reasoning and modeling using mathematics.	7.NS.1-3 N.Q.3	MP 2 MP 3 MP 4
Lesson 2: Interpreting Expressions	Students will begin this lesson by engaging in a "magic math" activity. This lesson will give students opportunities to explore and determine their understanding of expressions. They will be asked to consider, create and understand verbal representations of numbers and operations to symbolic representations using expressions. They will examine how symbolic manipulation of expressions affects values in real circumstances.	7.NS.3 A.SSE.1	MP 2 MP 6 MP 7 MP 8
Lesson 3: Reading and Evaluating	This lesson will give students an opportunity to fortify their understanding of interpreting and modifying expressions by analyzing symbolic manipulation of various expressions.	7.EE.2 A.SSE.1 A.SSE.2	MP 6 MP 7
Lesson 4: Constructing Equivalent Expressions	Students will begin this lesson by engaging in a real-life problem that encompasses some basic geometric concepts along with expression manipulation. This lesson will give students an opportunity to fortify their understanding of writing expressions.	A.SSE.1 A.SSE.2 A.SSE.3	MP 2 MP 3 MP 4 MP 7
Lesson 5: Constructing Equivalent Expressions	Students will begin this lesson by engaging in a task on developing expressions for a particular geometric pattern. This lesson will strengthen the ability of students to compare expressions presented in different forms and determine equivalency.	A-SSE.3 F-IF.8	MP 1 MP 3 MP 7 MP 8
Lesson 6: Distributive Law	Students will begin this lesson with an engaging activity that will lead to an understanding of rewriting and interpreting expressions using the distributive property.	A-SSE.1 A-SSE.2	MP 3 MP 4
Lesson 7: Formative Assessment Lesson	This lesson is intended to help teachers assess how well students are able to translate between words, symbols, tables, and area representations of algebraic expressions. It is designed to identify and support students who have difficulty in these concepts. (Shell Center Formative Assessment Lesson: Interpreting Algebraic Expressions)	A-SSE.1 A-SSE.2	MP 2 MP 7

Expressions
Lesson 1 of 7

Description:

Students will be introduced to the course using an estimation activity to develop conception of numbers and reinforce numeral operation fluency. It is also an entry activity into the course showcasing the explicit incorporation of math practices including problem solving, reasoning and modeling using mathematics.

Common Core State Standards Addressed:

- 7.NS.1-3: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Solve real-world and mathematical problems involving the four operations with rational numbers.
- N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Mathematical Practice Standard(s) Emphasized:

- MP2 - Reason abstractly and quantitatively.
- MP3 - Construct viable arguments and critique the reasoning of others.
- MP4 - Model with mathematics.

Sequence of Instruction	Activities Checklist

Engage

MATH PRACTICE 3

Play initial clip of Bucky the Badger. This content is from Dan Meyer's Three Act Math. http://mrmeyer.com/threeacts/buckythebadger/

Following the initial video clip (Act 1), ask students to guess how many push-ups Bucky had to perform in the course of the game. If a student responds by saying 83, then explain again how the number of pushups is calculated. After asking several students for approximations, split students up into groups of three or so students to further explore this question.

Explore

MATH PRACTICE 3

Ask the groups to construct viable arguments and critique the reasoning of others as they address the following questions:

INCLUDED IN THE STUDENT MANUAL

- About how many total push-ups do you think Bucky did during the game?
- Write down a number that you know is too high.
- Write down a number that you know is too low.
- What further information would you need to know in order to determine the exact number of total push-ups Bucky did in the course of the game?
- If you're Bucky, would you rather your team score their field goals at the start of the game or the end?
- What are some numbers of pushups that Bucky will never do in any game?

The key here is that the total depends on the order in which the touchdowns and field goals were scored, not just how many touchdowns and field goals were scored.

Explanation

Play clip that explains how many push-ups in total Bucky did (whether it is Bucky or more than one person is still a mystery!)

Address any questions or issues that may have come up as you observed the groups discuss the questions above.

Teacher's Note – A blog discussing the Bucky the Badger problem and the incorporation of problem solving and communication can be found at http://blog.mrmeyer.com/?p=13514. This may be used by the instructor to reflect on his/her own understandings and beliefs surrounding the Standards of Mathematical Practice.

Practice Together in Small Groups

MATH PRACTICE 2&3

No calculator should be used for Tasks 1 and 2. It is important to stress that in Task #2, students are asked to find approximate values. If students find themselves wanting or needing to use a calculator, give them a hand in reasoning abstractly and quantitatively through useful approximation strategies that help find a good estimate while being easy to compute.

INCLUDED IN THE STUDENT MANUAL

Task #1:

Use the fact that 13×17=221 to find the following:

a. 13×1.7
b. 130×17
c. 13×1700
d. 1.3×1.7
e. 2210÷13
f. 22100÷17 g. 221÷1.3

(http://illustrativemathematics.org/illustrations/272)

Commentary ***for the Teacher:***

This task is NOT an example of a task asking students to compute using the standard algorithms for multiplication and division because most people know what those kinds of problems look like. Instead, this task shows what kinds of reasoning and estimation strategies students need to develop in order to support their algorithmic computations.

Possible Solutions:

All these solutions use the associative and commutative properties of multiplication (explicitly or implicitly).

a. $13\times1.7=13\times(17\times0.1)=(13\times17)\times0.1$, so the product is one-tenth the product of 13 and 17. In other words:

$$13\times1.7=22.1$$

b. Since one of the factors is ten times one of the factors in 13×17, the product will be ten times as large as well:

$$130\times17=2210$$

c. $13\times1700=13\times(17\times100)=(13\times17)\times100$, so

$$13\times1700=22100$$

d. Since each of the factors is one tenth the corresponding factor in 13×17, the product will be one one-hundredth as large:

$$1.3\times1.7=2.21$$

e. $2210\div13=?$ is equivalent to $13\times?=2210$. Since the product is ten times as big and one of the factors is the same, the other factor must be ten times as big. So:

$$2210\div13=170$$

f. As in the previous problem, the product is 100 times as big, and since one factor is the same, the other factor must be 100 times as big:

$$22100\div17=1300$$

g. $221\div1.3=?$ is equivalent to $1.3\times?=221$. Since the product is the same size and one of the factors is one-tenth the size, the other factor must be ten times as big. So:

$$221\div1.3=170$$

INCLUDED IN THE STUDENT MANUAL

Task #2:

As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes but she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon.
(http://illustrativemathematics.org/illustrations/80)

a. Describe an estimate that Felicia might do in her head while driving to decide how many gallons of gas she needs to make it to the gas station at the other end.
b. Assuming she makes it, how much does Felicia spend per mile on the freeway?

Commentary ***for the Teacher:***

This task provides students the opportunity to make use of units to find the gas need. It also requires them to make some sensible approximations (e.g., 2.92 gallons is not a good answer to part (a)) and to recognize that Felicia's situation requires her to round up. Various answers to (a) are possible, depending on how much students think is a safe amount for Felicia to have left in the tank when she arrives at the gas station. The key point is for them to explain their choices. This task provides an opportunity for students to practice reasoning abstractly and quantitatively, and constructing viable arguments and critiquing the reasoning of others.

Possible Solution:

a. To estimate the amount of gas she needs, Felicia calculates the distance traveled at 70 mph for 1.25 hours. She might calculate:

$$70*1.25=70+0.25*70=70+17.5=87.5 \text{ miles.}$$

Since 1 gallon of gas will take her 30 miles, 3 gallons of gas will take her 90 miles, a little more than she needs. So she might figure that 3 gallons is enough.

Or, since she is driving, she might not feel like distracting herself by calculating 0.25x70 mentally, so she might replace 70 with 80, figuring that that will give her a larger distance than she needs. She calculates:

$$80*1.25=80+14*80=100.$$

So at 30 miles per gallon, 313 gallons will get her further than she needs to go and should be enough to get her to the gas station.

b. Since Felicia pays $3.50 for one gallon of gas, and one gallon of gas takes her 30 miles, it costs her $3.50 to travel 30 miles. Therefore, $3.50/30 miles = $0.121, meaning it costs Felicia 12 cents to travel each mile on the freeway.

Evaluate Understanding

Ask some students to share their strategies for solving some of the questions above. Be sure to emphasize good (and bad) approximation strategies, paying attention to units when appropriate, and reviewing the properties of operations when working with numerical expressions. Do NOT mention anything about PEMDAS. Students should use any (correct) order of operations, and the order of those operations should be a result of useful strategies. For example:

$$1.3x1.7 = (13)x(\frac{1}{10})x(17)x(\frac{1}{10}) = \frac{1}{100}x(13x17) = \frac{1}{100})x(221) = 2.21.$$

Here the strategy is using commutative and associative properties of multiplication rather than inventing a gimmicky trick with decimals that works in this one particular case. Reviewing and deepening the depth of understanding of these properties is crucial before moving on to working with algebraic expressions.

Closing Activity

MATH 4 PRACTICE

Still working in groups, ask the students to model with mathematics the following situation:

Let *x* denote the number of touchdowns Wisconsin scored in a game. Assuming the Wisconsin football team only scores touchdowns, write an algebraic expression

to represent the total number of pushups Bucky must do in a game in which *x* touchdowns are scored.

Independent Practice:

For this lesson the Independent Practice is the Closing Activity. Students may have started sharing ideas on how to find a general expression; have them complete the solution outside of class.

Resources/Instructional Materials Needed:

Computer/Projector

Video Clip: Three Act Math: Bucky the Badger — (http://mrmeyer.com/threeacts/buckythebadger/)

Notes:

Expressions
Lesson 2 of 7

Description:

Students will begin this lesson by engaging in a "magic math" activity. This lesson will give students opportunities to explore and determine their understanding of expressions. They will be asked to consider, create and understand verbal representations of numbers and operations to symbolic representations using expressions. They will examine how symbolic manipulation of expressions affects values in real circumstances.

Common Core State Standards Addressed:

- 7.NS.3: Solve real-world and mathematical problems involving the four operations with rational numbers.
- A.SSE.1: Interpret expressions that represent a quantity in terms of its context.

Mathematical Practice Standard(s) Emphasized:

- MP2 - Reason abstractly and quantitatively.
- MP6 - Attend to precision.
- MP7 - Look for and make use of structure.
- MP8 - Look for and express regularity in repeated reasoning.

Sequence of Instruction	Activities Checklist

Engage

Magic Math: Number Guess Introduction

- Have each student choose a number between one and 20 and write it down at the top of the Magic Math template provided in the Student Manual.
- Students should complete the following steps on their paper under their original number and write the instructions given in the second column.

INCLUDED IN THE STUDENT MANUAL

- Double your original number.
- Add 6.
- Divide by 2.
- Subtract the original number from the new number.
- Fold the paper once so your work/answer cannot be seen.

- Tell the students that you are going to come around and write your "guess" for their answer on the outside of the paper.
- Go around the room and write a 3 on the outside of their paper and turn your writing face down on the desk.
- Once you are finished writing your "guesses" on all of the papers ask them to look at your guess and if it matches their answer then have them raise their hand. (Hopefully all students will have their hand up at this point; however, if some do not just make note of it and address their calculation mistakes during the practice session).
- Make a "big deal" about how they must have all chosen the same number to start with.
- Ask two different students what numbers they chose and if you get two different starting numbers you might have to ask more than two students) then look puzzled.
- Ask the class as a whole, "How is it that <John> started with <4> and <Jane> started with <11>? They both performed the same operations on the two different numbers, but ended up with the same answer.
- Tell the students that you are going to give some time to discuss it.

Explore

Magic Math: Number Guess Exploration

- Have students pair up (one group of three if an odd number of students are present) with someone that chose a different start "/" original number.
- Have students discuss and write down on a sheet of paper their pair's understanding of why this process always results in a "3." Ask them to create a visual model of their thoughts. Have students look for and make use of structure as they create an expression representing all of the steps in the magic math number trick.
- Announce that if anyone did not get "3" as their answer (from a miscalculation), he/she should discuss the steps taken to arrive at the different solution with his/her partner. (Listen to the conversation surrounding the students' process and be prepared to ask guiding questions as necessary to help students find their errors in the event they are unable to locate the miscalculation.)
- Give time for student pairs to both quantitatively and abstractly reason through the problem and provide sound justification for their decisions.
- Walk around the room observing the explanations/models. Pay attention to the different correct approaches. Make note of any incorrect assumptions/processes.

Explanation

Magic Math: Number Guess Explanation

- Have one to three groups to communicate methods and solutions precisely to others through a report out of their processes. Try to select groups that have varying methods.
  - There is no need for the same exact process to be explained multiple times so choose pairs having some variations to share.
  - If you had one group that has an incorrect process you might sandwich them

between two correct groups. This way the students can solidify their thoughts with the first group. The second group will probably see their error and address it when presenting (but this provides a great opportunity for a group discussion about the reason for their miscalculation). Then the third group will provide reinforcement of the procedure.

- Leave students in their current groups and facilitate a whole group discussion about the process to include verbal, algebraic, and modeling representations.

Expressions

Magic Math (Explanations)

When I asked you to choose a number between one and 20, I had no real idea what you would choose. And in math, if we know a number exists but we don't know what particular number it is, then we use a variable or a symbol to represent that number. So let's go through this problem with ★ and *x* representing the chosen number.

Modeling Explanation	Verbal Process	Algebraic Explanation
★	Chosen Number	x
★ ★	Double it	$x \cdot 2 = 2x$
★ + ★ ● ● ● ● ●	Add 6 to it	$2x + 6$
★ + ★ + ● ● ● ● ●	Divide by 2	$\frac{2x + 6}{2} = x + 3$
~~★~~ + ● ● ● - ~~★~~	Subtract your original number	$x + 3 - x = 3$
● ● ●	Leaves 3	3

Practice Together in Small Groups

MATH 6&8 PRACTICE

Magic Math: Birthday Trick

- Have students work in pairs to complete the Math Magic: Birthday Trick from the Student Manual, taking turns with each student's own information.

INCLUDED IN THE STUDENT MANUAL

Do you believe that I can figure out your birthday by using simple math?

Get a calculator and ask your classmate to try the following:

a) Enter the month of his/her birth into the calculator. (Ex: enter 5 for May)
b) Multiply that number by 7.
c) Subtract 1 from that result.
d) Multiply that result by 13.
e) Add the day of birth. (Ex: For June 14th add 14)
f) Add 3.
g) Multiply by 11.
h) Subtract the month of birth.
i) Subtract the day of birth.
j) Divide by 10.
k) Add 11.
l) Divide by 100.

- Have the students look for and make use of repeated reasoning to model the process algebraically.
- Make sure that each of the members of the group can communicate the process that his/her pair used precisely.
- Have one student from each pair rotate to a different group.
- Have each student in the newly formed pairs explain to one another his/her model and the reasoning for each step.

Evaluate Understanding

Magic Math: Birthday Trick

- Monitor the different explanations in the groups and ask guiding questions aimed at correcting any misconceptions that may exist.

Closing Activity

Introduce Independent Practice

- Ask the students to use quantitative and abstract reasoning to create his/her own "magic trick." This should be at least a five step math process and should be represented through both verbal and algebraic representations. This is to be completed without the use of technology.
- Allow time for students to ask clarifying questions and summarize the independent practice task.

Independent Practice:

- Ask the students to use quantitative and abstract reasoning to create his/her own "magic trick." This should be at least a five step math process and should be represented through both verbal and algebraic representations.

Notes

SREB Readiness Courses

Transitioning to college and careers

Math Ready

Unit 1 . Expressions
Student Manual

Unit 1 . Expressions
Table of Contents

Restate the Bucky the Badger Problem in Your Own Words:

Construct a viable argument for the following:

About how many total push-ups do you think Bucky did during the game?

Write down a number that you know is too high.

Write down a number that you know is too low.

What further information would you need to know in order to determine the exact number of total push-ups Bucky did in the course?

If you're Bucky, would you rather your team score their field goals at the start of the game or the end?

What are some numbers of pushups that Bucky will never do in any game?

Task #1

Use the fact that 13×17=221 to find the following:

a. 13×1.7

b. 130×17

c. 13×1700

d. 1.3×1.7

e. 2210÷13

f. 22100÷17

g. 221÷1.3

(http://illustrativemathematics.org/illustrations/272)

Task #2

As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes, and she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon. (http://illustrativemathematics.org/illustrations/80)

a. Describe an estimate that Felicia might do in her head while driving to decide how many gallons of gas she needs to make it to the gas station at the other end.

b. Assuming she makes it, how much does Felicia spend per mile on the freeway?

Numbers and Operations

Magic Math

Numbers and Operations

Birthday Trick

Do you believe that I can figure out your birthday by using simple math?

Get a calculator and ask your classmate to try the following*:

a) Enter the month of his/her birth into the calculator. (Ex: enter 5 for May)

b) Multiply that number by 7. c)

Subtract 1 from that result. d)

Multiply that result by 13.

e) Add the day of birth. (Ex: For June 14th add 14)

f) Add 3.

g) Multiply by 11.

h) Subtract the month of birth. i)

Subtract the day of birth.

j) Divide by 10. k)

Add11.

l) Divide by 100.

**Your classmate must press equal (or enter) between every step.*