SREB

SREB Readiness Courses
Transitioning to High School

Ready for High School: Math

Southern Regional Education Board

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Introduction
SREB Readiness Courses

SREB has worked for over a decade with states to introduce policies and programs that address the issue of too many students graduating from high school not prepared for success in the college classroom or workplace. For states that wish to close the readiness gap, SREB has developed two sets of readiness courses in literacy and mathematics. The Ready for College courses are designed to give underprepared students a solid foundation for success in college and postsecondary training. The Ready for High School courses offer an earlier intervention, reaching underprepared students as they enter high school, which for many students is the most critical time in their education in determining future success.

Ready for High School Courses: Prepared for High School and Beyond

Research shows that between 70 and 80 percent of students who fail in the first year of high school will not graduate. By implementing courses in literacy and mathematics for underprepared students, schools can increase their ninth-grade retention rates and ultimately their high school graduation rates. To address the high failure and dropout rates that occur during the ninth grade, SREB has developed two courses, Ready for High School Literacy and Ready for High School Math, reaching underprepared students before they enter high school. These courses are being implemented in schools across the nation, and are available upon completion of training as a free download at http://www.sreb.org/ready-high-school.

Ready for High School: Math

This course emphasizes understanding of math concepts rather than just memorizing procedures. In Ready for High School Math, students learn why to use a certain formula or method to solve a problem. By engaging students in real-world applications, Ready for High School Math develops critical-thinking skills that students will use throughout their high school studies. The course consists of eight units, culminating in a capstone project.

The High School Math Ready course focuses on sixty-eight key readiness standards (listed on page VI) as well as ten Process Readiness Indicators needed for students to be ready to undertake high school-level courses aimed at making students college and career ready. The course addresses standards from throughout middle school and even earlier, including the topics of number sense, ratios and proportions, expressions and equations, algebra, functions, geometry, and statistics and probability agreed to as essential college and career-readiness standards for most students.

The math course consists of seven mandatory modules: The Number System, Ratio and Proportional Relationships, Probability and One Variable Statistics, Expressions, Equations and Inequalities, Geometry, Functions and Linear Relationships, and Systems of Equations.

An optional introductory unit, Operations on Fractions, is provided as well as an optional capstone project.

While this course covers the basics in math practices and reviews the procedural steps needed to be successful in math, it is designed to teach in a new, engaging way based heavily on conceptual teaching and learning. Each unit includes a "hook" at the beginning to engage students and pre-assess prior math experiences and understandings. The hook is followed by several days of tasks that delve deeply into math found in the Process Readiness Indicators and the lead headers of the readiness standards — focus, coherence and rigor. Each unit also includes at least one Formative Assessment Lesson appropriately placed within the unit, allowing the teacher to adapt instruction and learning prior to the conclusion of the unit.

Unit ½: Operations with Fractions

This optional introductory unit encourages a deeper understanding of order, comparison, and computation of fractions through exploration of different fractional models. This unit also introduces students to the type of thinking, questioning, and the general approach to instruction they will encounter throughout the remainder of the course.

Unit 1: The Number System

This unit is designed to solidify the understanding of the relationships between fractions, decimals and percents as well as explore scientific notation and irrational numbers.

Unit 2: Ratio and Proportional Relationships

This unit was designed to solidify students' understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems.

Unit 3: Probability and One Variable Statistics

This unit deals with calculating probability and understanding one-variable statistics. Students are asked to calculate the probability of compound events, describe the effects a single value can have on summary statistics, and utilize data to learn about specific populations.

Unit 4: Expressions, Equations and Inequalities

This unit deals the meaning and structure of expressions, equations and inequalities. Students use illustrations, drawings and models to represent and solve equations and inequalities to help them develop understanding of the solutions of equations and inequalities. AS effective strategies for solving equations and inequalities are developed, students will extend these strategies to literal equations.

Unit 5: Geometry

This unit will aid students in understanding how to draw, translate, and describe geometrical figures, understand congruence, use the Pythagorean Theorem and discuss relationships between different shapes in the context of real world mathematical problems.

Unit 6: Functions and Linear Relationships

In this unit, students will identify the characteristics that distinguish functions from relations and will identify functions as linear or non-linear. Students will investigate linear relationships in depth through tables, equations, and graphs. Students will

develop linear models for real-world situations. Students will relate slope as a rate of change and the y-intercept contextually to real-world problems.

Unit 7: Systems of Equations

This unit aids students in understanding the solution to a system of equations is the point of intersection when the equations are graph and that it contains the values that satisfies both equations. Students will be able to write and use a system of equations to solve a real-world problem.

Acknowledgements

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Math Ready Content Standards

Number Sense

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- NS.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

- NS.2: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
  a. Describe situations in which opposite quantities combine to make 0.
  b. Understand $p + q$ as the number located a distance $|q|$ from p,in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
  c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
  d. Apply properties of operations as strategies to add and subtract rational numbers.
- NS.3: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
  a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to
  b. satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
  c. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.
  d. Apply properties of operations as strategies to multiply and divide rational numbers.
  e. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Know that there are numbers that are not rational, and approximate them by rational numbers.

- NS.4: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

- NS.5: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions

Ratios and Proportions

Understand ratio concepts and use ratio reasoning to solve problems.

- RP.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- RP.2: Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* not equal to 0, and use rate language in the context of a ratio relationship.
- RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b. Solve unit rate problems including those involving unit pricing and constant speed.
  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.
  d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- RP.4: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- RP.5: Recognize and represent proportional relationships between quantities.
  a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  c. Represent proportional relationships by equations.
  d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
- RP.6: Use proportional relationships to solve multistep ratio and percent problems.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

- EE.1: Write, read, and evaluate expressions in which letters stand for numbers.
  a. Write expressions that record operations with numbers and with letters standing for numbers.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more

c. parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

d. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

- EE.2: Apply the properties of operations to generate equivalent expressions.
- EE.3: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Reason about and solve one-variable equations and inequalities.

- EE.4: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Represent and analyze quantitative relationships between dependent and independent variables.

- EE.5: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Use properties of operations to generate equivalent expressions.

- EE.6: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- EE.7: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies
- EE.8: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

  a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.

  b. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

  c. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Work with radicals and integer exponents.

- EE.9: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $32 \times 3 - 5 = 3 - 3 = \frac{1}{33} = \frac{1}{27}$.
- EE.10: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- EE.11: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

- EE.12: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- EE.13: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

- EE.14: Solve linear equations in one variable.
  a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
  c. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- EE.15: Analyze and solve pairs of simultaneous linear equations.
  a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their
  b. graphs, because points of intersection satisfy both equations simultaneously.
  c. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations.
  d. Solve simple cases by inspection.
  e. Solve real-world and mathematical problems leading to two linear equations in two variables.

Algebra

Interpret the structure of expressions

- A.1: Interpret expressions that represent a quantity in terms of its context
  a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems

- A.2: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  a. Factor a quadratic expression to reveal the zeros of the function it defines.
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  c. Use the properties of exponents to transform expressions for exponential functions.

Understand solving equations as a process of reasoning and explain the reasoning

- A.3: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable

- A.4: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Create equations that describe numbers or relationships

- A.5: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

Functions

Define, evaluate, and compare functions.

- F.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- F.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
- F.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

- F.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- F.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

- G.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- G.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- G.3: Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

- G.4: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- G.5: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Understand congruence and similarity using physical models, transparencies, or geometry software.

- G.6: Verify experimentally the properties of rotations, reflections, and translations:
  a. Lines are taken to lines, and line segments to line segments of the same length.
  b. Angles are taken to angles of the same measure.
  c. Parallel lines are taken to parallel lines.
- G.7: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- G.8: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- G.9: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- G.10: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Understand and apply the Pythagorean Theorem

- G.11: Explain a proof of the Pythagorean Theorem and its converse.

- G.12: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Develop understanding of statistical variability.

- SP.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- SP.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- SP.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

- SP.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- SP.5: Summarize numerical data sets in relation to their context, such as by:
  a. Reporting the number of observations.
  b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
  d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Use random sampling to draw inferences about a population.

- SP.6: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- SP.7: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

- SP.8: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

- SP.9: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

- SP.10: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
- SP.11: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
  a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine
  b. probabilities of events.
  c. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
- SP.12: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
  a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
  b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
  c. Design and use a simulation to generate frequencies for compound events.

Investigate patterns of association in bivariate data.

- SP.13: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- SP.14: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- SP.15: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- SP.16: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Optional Unit ½:

Number Operations

Use equivalent fractions as a strategy to add and subtract fractions.

- NO.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
- NO.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- NO.3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- NO.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
  a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.
  b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

SREB Process Readiness Indicators:

- PRI 1: Make sense of problems and persevere in solving through reasoning and exploration.
- PRI 2: Reason abstractly and quantitatively by using multiple forms of representations to make sense of and understand mathematics.
- PRI 3: Describe and justify mathematical understandings by constructing viable arguments, critiquing the reasoning of others and engaging in meaningful mathematical discourse.
- PRI 4: Contextualize mathematical ideas by connecting them to real-world situations. Model with mathematics.
- PRI 5: Use appropriate tools strategically to support thinking and problem solving.
- PRI 6: Attend to precision.
- PRI 7: Look for and make use of patterns and structure.
- PRI 8: Look for and express regularity in repeated reasoning.
- PRI 9: Demonstrate flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
- PRI 10: Reflect on mistakes and misconceptions to improve mathematical understanding.